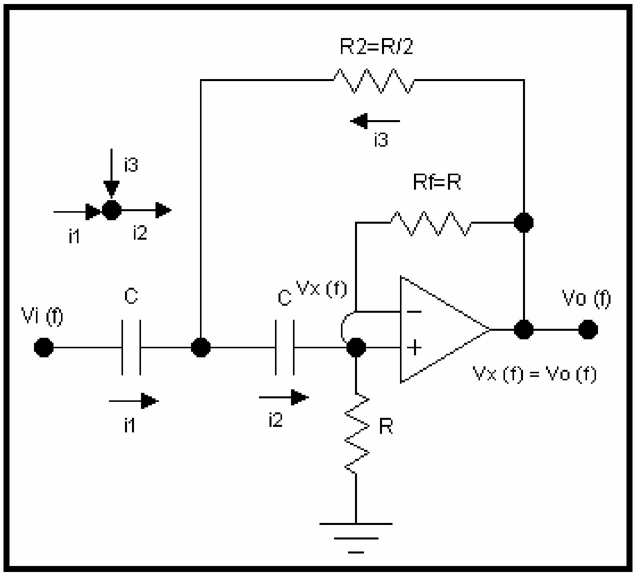
FILTRO PASA ALTAS DE –40 db / dec



a. Hallar $A_v(w) = f(R, C, w)$

$$V_x(w) = \frac{R}{R+Z_c} V_y(w) \quad \rightarrow \quad V_x(w) = \frac{R}{R+\dfrac{1}{jwC}} V_y(w) \quad \rightarrow$$

$$V_x(w) = \frac{R}{\dfrac{jwRC+1}{jwC}} V_y(w)$$

$$V_x(w) = \frac{jwRC}{jwRC+1} V_y(w) \quad \rightarrow \quad V_x(w) = V_o(w) \quad \rightarrow$$

$$\boxed{V_o(w) = \frac{jwRC}{jwRC+1} V_y(w) \qquad (1)}$$

$$i_2 = i_1 + i_3$$

$$\frac{V_y}{Z_c} - \frac{V_x}{Z_c} = \frac{V_i}{Z_c} - \frac{V_y}{Z_c} + \frac{V_x}{\frac{R}{2}} - \frac{V_y}{\frac{R}{2}} \quad \rightarrow \quad \frac{V_y}{Z_c} - \frac{V_x}{Z_c} = \frac{V_i}{Z_c} - \frac{V_y}{Z_c} + \frac{2V_x}{R} - \frac{2V_y}{R}$$

$$\frac{V_y}{Z_c} + \frac{V_y}{Z_c} + \frac{2V_y}{R} = \frac{V_i}{Z_c} + \frac{2V_x}{R} + \frac{V_x}{Z_c} \quad \rightarrow \quad V_y\left(\frac{2}{Z_c} + \frac{2}{R}\right) = \frac{RV_i + 2Z_cV_x + RV_x}{RZ_c}$$

$$V_y(w)=\frac{\dfrac{RV_i+2Z_cV_x+RV_x}{RZ_c}}{\dfrac{2R+2Z_c}{RZ_c}} \quad \rightarrow \quad V_y(w)=\frac{RV_i+2Z_cV_x+RV_x}{2R+2Z_c}$$

$$V_y(w)=\frac{RV_i+\dfrac{2V_x}{jwC}+RV_x}{2R+\dfrac{2}{jwC}} \quad \rightarrow \quad V_y(w)=\frac{\dfrac{jwRCV_i+2V_x+jwRCV_x}{jwC}}{\dfrac{2jwRC+2}{jwC}}$$

$$V_y(w)=\frac{jwRCV_i+2V_x+jwRCV_x}{2jwRC+2} \quad \rightarrow \quad V_x(w)=V_o(w)$$

$$\boxed{V_y(w)=\frac{jwRCV_i+2V_o+jwRCV_o}{2jwRC+2} \qquad (2)}$$

Reemplazando 2 en 1

$$V_o(w)=\frac{jwRC}{jwRC+1}\left[\frac{jwRCV_i+2V_o+jwRCV_o}{2jwRC+2}\right]$$

$$V_o(w)=\frac{(jwRC)^2V_i+2jwRCV_o+(jwRC)^2V_o}{2(jwRC+1)^2}$$

$$2(jwRC+1)^2V_o(w)-2jwRCV_o-(jwRC)^2V_o=(jwRC)^2V_i$$

$$V_o(w)\left[2(jwRC+1)^2-2jwRC-(jwRC)^2\right]=(jwRC)^2V_i$$

$$V_o(w)=\frac{-w^2R^2C^2V_i}{2\left(-w^2R^2C^2+2jwRC+1\right)-2jwRC+w^2R^2C^2}$$

$$V_o(w)=\frac{-w^2R^2C^2}{-2w^2R^2C^2+4jwRC+2-2jwRC+w^2R^2C^2}V_i$$

$$\frac{V_o(w)}{V_i(w)}=\frac{-w^2R^2C^2}{-w^2R^2C^2+2jwRC+2} \quad \rightarrow \quad \boxed{\left|A_v(w)\right|=\frac{-w^2R^2C^2}{\sqrt{\left(2-w^2R^2C^2\right)^2+(2wRC)^2}}}$$

$$\lim_{w\to\infty}\left|A_v(w)\right|=\frac{\infty}{\infty}=1$$ (Las frecuencias altas pasan)

$$\lim_{w\to 0}\left|A_v(w)\right| = \frac{0}{\sqrt{4}} = \frac{0}{2} = 0$$ (Las frecuencias bajas no pasan)

b. Hallar $w_c \;\; y \;\; f_c$

$$\frac{1}{\sqrt{2}} = \frac{-w_c{}^2R^2C^2}{\sqrt{\left(2-w_c{}^2R^2C^2\right)^2+\left(2w_cRC\right)^2}} \quad \rightarrow$$

$$\left(\sqrt{\left(2-w_c{}^2R^2C^2\right)^2+\left(2w_cRC\right)^2} = -w_c{}^2R^2C^2\sqrt{2}\right)^2$$

$$\left(2-w_c{}^2R^2C^2\right)^2+\left(2w_cRC\right)^2 = 2w_c{}^4R^4C^4$$

$$4-4w_c{}^2R^2C^2+w_c{}^4R^4C^4+4w_c{}^2R^2C^2 = 2w_c{}^4R^4C^4$$

$$4+w_c{}^4R^4C^4 = 2w_c{}^4R^4C^4 \quad \rightarrow \quad w_c{}^4R^4C^4 = 4 \quad \rightarrow \quad w_c{}^4 = \frac{4}{R^4C^4}$$

$$\sqrt[4]{w_c{}^4} = \sqrt[4]{\frac{4}{R^4C^4}} \quad \rightarrow \quad \boxed{w_c = \frac{\sqrt[4]{4}}{RC} = \frac{\sqrt{2}}{RC}} \quad \rightarrow \quad 2\pi f_c = \frac{\sqrt{2}}{RC} \rightarrow \quad \boxed{f_c = \frac{\sqrt{2}}{2\pi RC}}$$

c. Hallar $\left|A_v(f)\right| = f(f, f_c)$

$$A_v(w) = \frac{-w^2\left(\frac{\sqrt{2}}{w_c}\right)^2}{\sqrt{\left(2-w^2\left(\frac{\sqrt{2}}{w_c}\right)^2\right)^2+\left(2w\frac{\sqrt{2}}{w_c}\right)^2}} \quad \rightarrow \quad A_v(w) = \frac{-2\left(\frac{w}{w_c}\right)^2}{\sqrt{\left(2-2\left(\frac{w}{w_c}\right)^2\right)^2+8\left(\frac{w}{w_c}\right)^2}}$$

$$A_v(w) = \frac{-2\left(\frac{2\pi f}{2\pi f_c}\right)^2}{\sqrt{\left(2-2\left(\frac{2\pi f}{2\pi f_c}\right)^2\right)^2+8\left(\frac{2\pi f}{2\pi f_c}\right)^2}} \quad \rightarrow \quad \boxed{A_v(w) = \frac{-2\left(\frac{f}{f_c}\right)^2}{\sqrt{\left(2-2\left(\frac{f}{f_c}\right)^2\right)^2+8\left(\frac{f}{f_c}\right)^2}}}$$

d. Dibujar $\left|A_v(f)\right| \;\; Vs \;\; f$

$$A_v(w) = \frac{-2\left(\frac{f}{f_c}\right)^2}{\sqrt{\left(2-2\left(\frac{f}{f_c}\right)^2\right)^2 + 8\left(\frac{f}{f_c}\right)^2}}$$

$$A_v = 20\log\left(\frac{V_0}{V_i}\right)$$

f	$\left\|A_v(f)\right\|$	$\left\|A_v(f)\right\|\ db$
0	0	0
0.1fc	0.01	-40
0.5fc	0.24	-12
0.8 fc	0.54	-5
fc	0.71	-3
2 fc	0.97	0
4 fc	1	0
6 fc	1	0
8 fc	1	0
10 fc	1	0