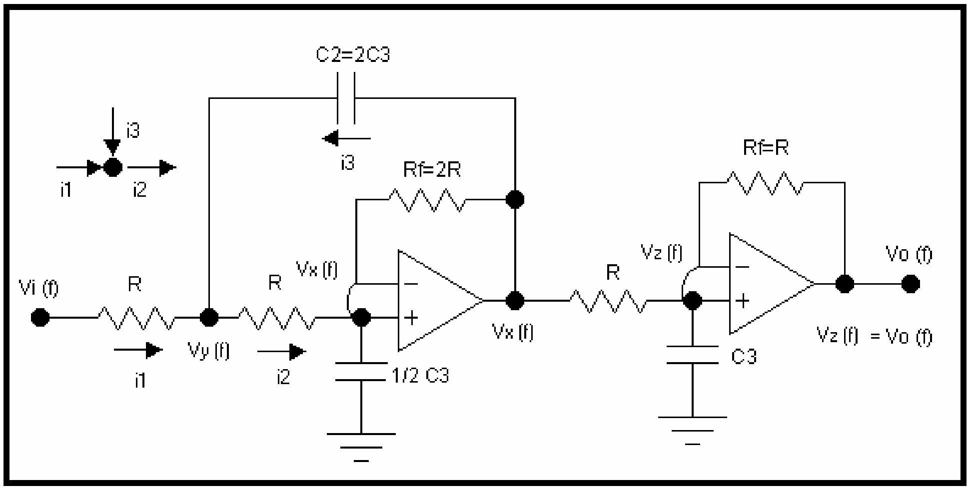
FILTRO PASABAJAS DE –60 db / dec



a. Hallar $A_V(w) = f(R, C, w)$

$$V_x = \frac{Z_{c1}}{R + Z_{c1}} V_y \quad \rightarrow \quad C_1 = \frac{1}{2} C_3 \quad \rightarrow \quad V_x = \frac{\dfrac{2}{jwC_3}}{R + \dfrac{2}{jwC_3}} V_y$$

$$V_x = \frac{\dfrac{2}{jwC_3}}{\dfrac{jwRC_3 + 2}{jwC_3}} V_y \quad \rightarrow \quad \boxed{V_x = \frac{2}{jwRC_3 + 2} V_y \quad (1)}$$

$$V_z = \frac{Z_{c3}}{R + Z_{c3}} V_y \quad \rightarrow \quad V_z = \frac{\dfrac{1}{jwC_3}}{R + \dfrac{1}{jwC_3}} V_y \quad \rightarrow \quad V_z = \frac{\dfrac{1}{jwC_3}}{R + \dfrac{1}{jwC_3}} V_x$$

$$V_z = \frac{\dfrac{1}{jwC_3}}{\dfrac{jwRC_3 + 1}{jwC_3}} V_x \quad \rightarrow \quad \boxed{V_z = \frac{1}{jwRC_3 + 1} V_x \quad (2)}$$

$$i2 = i1 + i3$$

$$\frac{V_y}{R}-\frac{V_x}{R}=\frac{V_i}{R}-\frac{V_y}{R}+\frac{V_x}{Z_{c2}}-\frac{V_y}{Z_{c2}} \quad \rightarrow \quad \frac{V_y}{R}+\frac{V_y}{R}+\frac{V_y}{Z_{c2}}=\frac{V_i}{R}+\frac{V_x}{R}+\frac{V_x}{Z_{c2}}$$

$$V_y\left(\frac{2}{R}+\frac{1}{Z_{c2}}\right)=\frac{Z_{c2}V_i+RV_x+V_xZ_{c2}}{RZ_{c2}} \quad \rightarrow \quad V_y=\frac{\frac{Z_{c2}V_i+RV_x+V_xZ_{c2}}{RZ_{c2}}}{\frac{2Z_{c2}+R}{RZ_{c2}}}$$

$$V_y(w)=\frac{Z_{c2}V_i+RV_x+V_xZ_{c2}}{2Z_{c2}+R}$$, donde:

$$Z_{c2}=\frac{1}{jwC_2},\ pero\ como\ C_2=2C_3,\ entonces:\ Z_{c2}=\frac{1}{2jwC_3}$$

$$V_y(w)=\frac{\frac{1}{2jwC_3}V_i+RV_x+\frac{1}{2jwC_3}V_x}{\frac{2}{2jwC_3}+R} \quad \rightarrow \quad V_y(w)=\frac{\frac{V_i+2jwRC_3V_x+V_x}{2jwC_3}}{\frac{2+2jwRC_3}{2jwC_3}}$$

$$V_y(w)=\frac{V_i+2jwRC_3V_x+V_x}{2+2jwRC_3} \quad (3)$$, reemplazando 1 en 3:

$$V_y(w)=\frac{V_i+2jwRC_3\left(\frac{2}{jwRC_3+2}V_y\right)+\frac{2}{jwRC_3+2}V_y}{2+2jwRC_3}$$

$$V_y(w)=\frac{V_i+\frac{4jwRC_3}{jwRC_3+2}V_y+\frac{2}{jwRC_3+2}V_y}{2+2jwRC_3}$$

$$V_y(w)=\frac{\frac{V_i(jwRC_3+2)+4jwRC_3V_y+2V_y}{jwRC_3+2}}{2+2jwRC_3}$$

$$V_y(w)=\frac{V_i(jwRC_3+2)+4jwRC_3V_y+2V_y}{(2+2jwRC_3)(jwRC_3+2)}$$

$$(2+2jwRC_3)(jwRC_3+2)V_y(w)-4jwRC_3V_y-2V_y=V_i(jwRC_3+2)$$

$$V_y(w)\left((2+2jwRC_3)(jwRC_3+2)-4jwRC_3-2\right)=V_i(jwRC_3+2)$$

$$V_y(w)=\frac{jwRC_3+2}{(2+2jwRC_3)(jwRC_3+2)-4jwRC_3-2}V_i \quad (4)$$, reemplazando 4 en 1

$$V_x(w)=\frac{2}{jwRC_3+2}\left[\frac{jwRC_3+2}{(2+2jwRC_3)(jwRC_3+2)-4jwRC_3-2}V_i\right]$$

$$V_x(w)=\frac{2}{(2+2jwRC_3)(jwRC_3+2)-4jwRC_3-2}V_i \quad (5)$$

Reemplazo 5 en 2

$$V_z=\frac{1}{jwRC_3+1}\left[\frac{2}{(2+2jwRC_3)(jwRC_3+2)-4jwRC_3-2}V_i\right]$$

$$V_z=\frac{2}{(jwRC_3+1)(2+2jwRC_3)(jwRC_3+2)-(jwRC_3+1)(4jwRC_3+2)}V_i$$

$$(jwRC_3+1)(2+2jwRC_3)(jwRC_3+2)$$
$$=(2jwRC_3+2-2w^2R^2C_3{}^2+2jwRC_3)(jwRC_3+2)$$
$$=-2w^2R^2C_3{}^2+2jwRC_3-2jw^3R^3C_3{}^3-2w^2R^2C_3{}^2+4jwRC_3+4-4w^2R^2C_3{}^2+4jwRC_3$$
$$=-2jw^3R^3C_3{}^3-8w^2R^2C_3{}^2+10jwRC_3+4$$

$$(jwRC_3+1)(4jwRC_3+2)$$
$$=-4w^2R^2C_3{}^2+4jwRC_3+2jwRC_3+2$$
$$=-4w^2R^2C_3{}^2+6jwRC_3+2$$

en donde:

$$(jwRC_3+1)(2+2jwRC_3)(jwRC_3+2)-(jwRC_3+1)(4jwRC_3+2)$$
$$=-2jw^3R^3C_3{}^3-8w^2R^2C_3{}^2+10jwRC_3+4-(-4w^2R^2C_3{}^2+6jwRC_3+2)$$
$$=-2jw^3R^3C_3{}^3-8w^2R^2C_3{}^2+10jwRC_3+4+4w^2R^2C_3{}^2-6jwRC_3-2$$
$$=-2jw^3R^3C_3{}^3-4w^2R^2C_3{}^2+4jwRC_3+2$$

por lo tanto:

$$V_z(w)=\frac{2}{(jwRC_3+1)(2+2jwRC_3)(jwRC_3+2)-(jwRC_3+1)(4jwRC_3+2)}V_i(w)$$

$$V_z(w)=\frac{2}{=-2jw^3R^3C_3^{\ 3}-4w^2R^2C_3^{\ 2}+4jwRC_3+2}V_i(w)$$

$$V_z(w)=\frac{1}{-jw^3R^3C_3^{\ 3}-2w^2R^2C_3^{\ 2}+2jwRC_3+1}V_i(w)$$

$$A_v(w)=\frac{1}{-jw^3R^3C_3^{\ 3}-2w^2R^2C_3^{\ 2}+2jwRC_3+1}$$

$$\boxed{\left|A_v(w)\right|=\frac{1}{\sqrt{\left(1-2w^2R^2C_3^{\ 2}\right)^2+\left(2wRC_3-w^3R^3C_3^{\ 3}\right)^2}}}$$

$$\lim_{w\to\infty}\left|A_v(w)\right|=\frac{1}{\sqrt{\infty^2}}=\frac{1}{\infty}=0$$ (Las frecuencias altas no pasan)

$$\lim_{w\to 0}\left|A_v(w)\right|=\frac{1}{\sqrt{1}}=\frac{1}{1}=1$$ (Las frecuencias bajas pasan)

b. Hallar w_c y f_c

$$\frac{1}{\sqrt{2}}=\frac{1}{\sqrt{\left(1-2w_c^{\ 2}R^2C_3^{\ 2}\right)^2+\left(2w_cRC_3-w_c^{\ 3}R^3C_3^{\ 3}\right)^2}}$$

$$\left(\sqrt{\left(1-2w_c^{\ 2}R^2C_3^{\ 2}\right)^2+\left(2w_cRC_3-w_c^{\ 3}R^3C_3^{\ 3}\right)^2}=\sqrt{2}\right)^2$$

$$\left(1-2w_c^{\ 2}R^2C_3^{\ 2}\right)^2+\left(2w_cRC_3-w_c^{\ 3}R^3C_3^{\ 3}\right)^2=2$$

$$1-4w_c^{\ 2}R^2C_3^{\ 2}+4w_c^{\ 4}R^4C_3^{\ 4}+4w_c^{\ 2}R^2C_3^{\ 2}-4w_c^{\ 4}R^4C_3^{\ 4}+w_c^{\ 6}R^6C_3^{\ 6}=2$$

Simplificando:

$$w_c^{\ 6}R^6C_3^{\ 6}=1 \quad\to\quad w_c^{\ 6}=\frac{1}{R^6C_3^{\ 6}} \to \quad \sqrt[6]{w_c^{\ 6}}=\sqrt[6]{\frac{1}{R^6C_3^{\ 6}}}$$

$$\boxed{w_c=\frac{1}{RC_3}} \quad\to\quad 2\pi f_c=\frac{1}{RC_3} \quad\to\quad \boxed{f_c=\frac{1}{2\pi RC_3}}$$

c. Hallar $|A_v(f)| = f(f, f_c)$

$$A_v(w) = \frac{1}{\sqrt{\left(1-2w^2R^2C_3^{\,2}\right)^2 + \left(2wRC_3 - w^3R^3C_3^{\,3}\right)^2}}$$

$$A_v(w) = \frac{1}{\sqrt{\left(1-2w^2\left(\frac{1}{w_c}\right)^2\right)^2 + \left(2w\left(\frac{1}{w_c}\right) - w^3\left(\frac{1}{w_c^{\,3}}\right)\right)^2}}$$

$$A_v(w) = \frac{1}{\sqrt{\left(1-2\left(\frac{w}{w_c}\right)^2\right)^2 + \left(2\frac{w}{w_c} - \frac{w^3}{w_c^{\,3}}\right)^2}} \quad \rightarrow \quad w = 2\pi f \quad \text{y} \quad w_c = 2\pi f_c$$

$$A_v(w) = \frac{1}{\sqrt{\left(1-2\left(\frac{2\pi f}{2\pi f_c}\right)^2\right)^2 + \left(2\frac{2\pi f}{2\pi f_c} - \frac{2\pi f^3}{2\pi f_c^{\,3}}\right)^2}}$$

$$\boxed{A_v(w) = \frac{1}{\sqrt{\left(1-2\left(\frac{f}{f_c}\right)^2\right)^2 + \left(2\frac{f}{f_c} - \frac{f^3}{f_c^{\,3}}\right)^2}}}$$

d. Dibujar $|A_v(f)|\ Vs\ f$

$$\boxed{A_v(w) = \frac{1}{\sqrt{\left(1-2\left(\frac{f}{f_c}\right)^2\right)^2 + \left(2\frac{f}{f_c} - \frac{f^3}{f_c^{\,3}}\right)^2}}}$$

$$\boxed{A_v = 20\log\left(\frac{V_0}{V_i}\right)}$$

f	$\lvert A_v(f) \rvert$	$\lvert A_v(f) \rvert\ db$
0	1	0
0.1fc	1	0
0.5fc	1	0
0.8 fc	0.9	-1
fc	0.7	-3
2 fc	0.1	-18
4 fc	0	-36
6 fc	0	-47
8 fc	0	-54
10 fc	0	-60