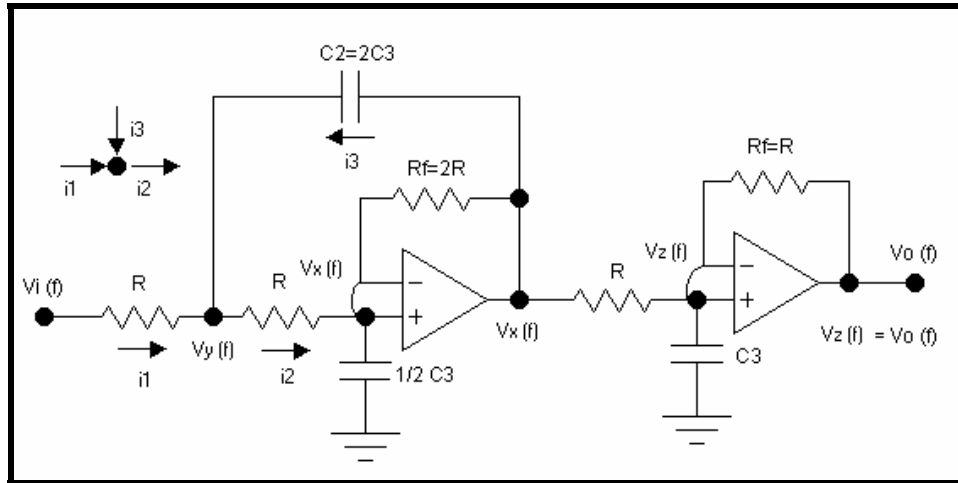


FILTRO PASABAJAS DE -60 db/dec



a. Hallar  $A_v(\omega) = f(R, C, \omega)$

$$V_x = \frac{Z_{c1}}{R + Z_{c1}} V_y \rightarrow C_1 = \frac{1}{2} C_3 \rightarrow V_x = \frac{\frac{2}{j\omega C_3}}{R + \frac{2}{j\omega C_3}} V_y$$

$$V_x = \frac{\frac{2}{j\omega C_3}}{\frac{j\omega R C_3 + 2}{j\omega C_3}} V_y \rightarrow \boxed{V_x = \frac{2}{j\omega R C_3 + 2} V_y \quad (1)}$$

$$V_z = \frac{Z_{c3}}{R + Z_{c3}} V_x \rightarrow V_z = \frac{\frac{1}{j\omega C_3}}{R + \frac{1}{j\omega C_3}} V_x \rightarrow V_z = \frac{\frac{1}{j\omega C_3}}{R + \frac{1}{j\omega C_3}} V_x$$

$$V_z = \frac{\frac{1}{j\omega C_3}}{\frac{j\omega R C_3 + 1}{j\omega C_3}} V_x \rightarrow \boxed{V_z = \frac{1}{j\omega R C_3 + 1} V_x \quad (2)}$$

$$i_2 = i_1 + i_3$$



$$\frac{V_y}{R} - \frac{V_x}{R} = \frac{V_i}{R} - \frac{V_y}{R} + \frac{V_x}{Z_{c2}} - \frac{V_y}{Z_{c2}} \quad \rightarrow \quad \frac{V_y}{R} + \frac{V_y}{R} + \frac{V_y}{Z_{c2}} = \frac{V_i}{R} + \frac{V_x}{R} + \frac{V_x}{Z_{c2}}$$

$$V_y \left( \frac{2}{R} + \frac{1}{Z_{c2}} \right) = \frac{Z_{c2}V_i + RV_x + V_xZ_{c2}}{RZ_{c2}} \quad \rightarrow \quad V_y = \frac{\frac{Z_{c2}V_i + RV_x + V_xZ_{c2}}{RZ_{c2}}}{\frac{2Z_{c2} + R}{RZ_{c2}}}$$

$$V_y(w) = \frac{Z_{c2}V_i + RV_x + V_xZ_{c2}}{2Z_{c2} + R}, \text{ donde:}$$

$$Z_{c2} = \frac{1}{jwC_2}, \text{ pero como } C_2 = 2C_3, \text{ entonces: } Z_{c2} = \frac{1}{2jwC_3}$$

$$V_y(w) = \frac{\frac{1}{2jwC_3}V_i + RV_x + \frac{1}{2jwC_3}V_x}{\frac{2}{2jwC_3} + R} \quad \rightarrow \quad V_y(w) = \frac{\frac{V_i + 2jwRC_3V_x + V_x}{2jwC_3}}{\frac{2 + 2jwRC_3}{2jwC_3}}$$

$$V_y(w) = \frac{V_i + 2jwRC_3V_x + V_x}{2 + 2jwRC_3} \quad (3), \text{ reemplazando 1 en 3:}$$

$$V_y(w) = \frac{V_i + 2jwRC_3 \left( \frac{2}{jwRC_3 + 2} V_y \right) + \frac{2}{jwRC_3 + 2} V_y}{2 + 2jwRC_3}$$

$$V_y(w) = \frac{V_i + \frac{4jwRC_3}{jwRC_3 + 2} V_y + \frac{2}{jwRC_3 + 2} V_y}{2 + 2jwRC_3}$$

$$V_y(w) = \frac{V_i(jwRC_3 + 2) + 4jwRC_3V_y + 2V_y}{2 + 2jwRC_3}$$

$$V_y(w) = \frac{V_i(jwRC_3 + 2) + 4jwRC_3V_y + 2V_y}{(2 + 2jwRC_3)(jwRC_3 + 2)}$$



$$(2 + 2jwRC_3)(jwRC_3 + 2)V_y(w) - 4jwRC_3V_y - 2V_y = V_i(jwRC_3 + 2)$$

$$V_y(w)((2 + 2jwRC_3)(jwRC_3 + 2) - 4jwRC_3 - 2) = V_i(jwRC_3 + 2)$$

$$V_y(w) = \frac{jwRC_3 + 2}{(2 + 2jwRC_3)(jwRC_3 + 2) - 4jwRC_3 - 2} V_i \quad (4), \text{ reemplazando 4 en 1}$$

$$V_x(w) = \frac{2}{jwRC_3 + 2} \left[ \frac{jwRC_3 + 2}{(2 + 2jwRC_3)(jwRC_3 + 2) - 4jwRC_3 - 2} V_i \right]$$

$$V_x(w) = \frac{2}{(2 + 2jwRC_3)(jwRC_3 + 2) - 4jwRC_3 - 2} V_i \quad (5)$$

Reemplazo 5 en 2

$$V_z = \frac{1}{jwRC_3 + 1} \left[ \frac{2}{(2 + 2jwRC_3)(jwRC_3 + 2) - 4jwRC_3 - 2} V_i \right]$$

$$V_z = \frac{2}{(jwRC_3 + 1)(2 + 2jwRC_3)(jwRC_3 + 2) - (jwRC_3 + 1)(4jwRC_3 + 2)} V_i$$

$$\begin{aligned} & (jwRC_3 + 1)(2 + 2jwRC_3)(jwRC_3 + 2) \\ &= (2jwRC_3 + 2 - 2w^2R^2C_3^2 + 2jwRC_3)(jwRC_3 + 2) \\ &= -2w^2R^2C_3^2 + 2jwRC_3 - 2jw^3R^3C_3^3 - 2w^2R^2C_3^2 + 4jwRC_3 + 4 - 4w^2R^2C_3^2 + 4jwRC_3 \\ &= -2jw^3R^3C_3^3 - 8w^2R^2C_3^2 + 10jwRC_3 + 4 \end{aligned}$$

$$\begin{aligned} & (jwRC_3 + 1)(4jwRC_3 + 2) \\ &= -4w^2R^2C_3^2 + 4jwRC_3 + 2jwRC_3 + 2 \\ &= -4w^2R^2C_3^2 + 6jwRC_3 + 2 \end{aligned}$$

en donde:

$$\begin{aligned} & (jwRC_3 + 1)(2 + 2jwRC_3)(jwRC_3 + 2) - (jwRC_3 + 1)(4jwRC_3 + 2) \\ &= -2jw^3R^3C_3^3 - 8w^2R^2C_3^2 + 10jwRC_3 + 4 - (-4w^2R^2C_3^2 + 6jwRC_3 + 2) \\ &= -2jw^3R^3C_3^3 - 8w^2R^2C_3^2 + 10jwRC_3 + 4 + 4w^2R^2C_3^2 - 6jwRC_3 - 2 \\ &= -2jw^3R^3C_3^3 - 4w^2R^2C_3^2 + 4jwRC_3 + 2 \end{aligned}$$



por lo tanto:

$$V_z(w) = \frac{2}{(jwRC_3 + 1)(2 + 2jwRC_3)(jwRC_3 + 2) - (jwRC_3 + 1)(4jwRC_3 + 2)} V_i(w)$$

$$V_z(w) = \frac{2}{-2jw^3R^3C_3^3 - 4w^2R^2C_3^2 + 4jwRC_3 + 2} V_i(w)$$

$$V_z(w) = \frac{1}{-jw^3R^3C_3^3 - 2w^2R^2C_3^2 + 2jwRC_3 + 1} V_i(w)$$

$$A_v(w) = \frac{1}{-jw^3R^3C_3^3 - 2w^2R^2C_3^2 + 2jwRC_3 + 1}$$

$$|A_v(w)| = \frac{1}{\sqrt{(1 - 2w^2R^2C_3^2)^2 + (2wRC_3 - w^3R^3C_3^3)^2}}$$

$$\lim_{w \rightarrow \infty} |A_v(w)| = \frac{1}{\sqrt{\infty^2}} = \frac{1}{\infty} = 0 \quad (\text{Las frecuencias altas no pasan})$$

$$\lim_{w \rightarrow 0} |A_v(w)| = \frac{1}{\sqrt{1}} = \frac{1}{1} = 1 \quad (\text{Las frecuencias bajas pasan})$$

**b. Hallar  $w_c$  y  $f_c$**

$$\frac{1}{\sqrt{2}} = \frac{1}{\sqrt{(1 - 2w_c^2R^2C_3^2)^2 + (2w_cRC_3 - w_c^3R^3C_3^3)^2}}$$

$$\left( \sqrt{(1 - 2w_c^2R^2C_3^2)^2 + (2w_cRC_3 - w_c^3R^3C_3^3)^2} = \sqrt{2} \right)^2$$

$$(1 - 2w_c^2R^2C_3^2)^2 + (2w_cRC_3 - w_c^3R^3C_3^3)^2 = 2$$

$$1 - 4w_c^2R^2C_3^2 + 4w_c^4R^4C_3^4 + 4w_c^2R^2C_3^2 - 4w_c^4R^4C_3^4 + w_c^6R^6C_3^6 = 2$$

Simplificando:

$$w_c^6R^6C_3^6 = 1 \quad \rightarrow \quad w_c^6 = \frac{1}{R^6C_3^6} \quad \rightarrow \quad \sqrt[6]{w_c^6} = \sqrt[6]{\frac{1}{R^6C_3^6}}$$

$$\boxed{w_c = \frac{1}{RC_3}}$$

$$\rightarrow \quad 2\pi f_c = \frac{1}{RC_3}$$

$$\rightarrow \quad \boxed{f_c = \frac{1}{2\pi RC_3}}$$

c. Hallar  $|A_v(f)| = f(f, f_c)$

$$A_v(\omega) = \frac{1}{\sqrt{\left(1 - 2\omega^2 R^2 C_3^2\right)^2 + \left(2\omega R C_3 - \omega^3 R^3 C_3^3\right)^2}}$$

$$A_v(\omega) = \frac{1}{\sqrt{\left(1 - 2\omega^2 \left(\frac{1}{\omega_c}\right)^2\right)^2 + \left(2\omega \left(\frac{1}{\omega_c}\right) - \omega^3 \left(\frac{1}{\omega_c^3}\right)\right)^2}}$$

$$A_v(\omega) = \frac{1}{\sqrt{\left(1 - 2\left(\frac{\omega}{\omega_c}\right)^2\right)^2 + \left(2\frac{\omega}{\omega_c} - \frac{\omega^3}{\omega_c^3}\right)^2}} \rightarrow \omega = 2\pi f \quad \text{y} \quad \omega_c = 2\pi f_c$$

$$A_v(\omega) = \frac{1}{\sqrt{\left(1 - 2\left(\frac{2\pi f}{2\pi f_c}\right)^2\right)^2 + \left(2\frac{2\pi f}{2\pi f_c} - \frac{2\pi f^3}{2\pi f_c^3}\right)^2}}$$

$$A_v(\omega) = \frac{1}{\sqrt{\left(1 - 2\left(\frac{f}{f_c}\right)^2\right)^2 + \left(2\frac{f}{f_c} - \frac{f^3}{f_c^3}\right)^2}}$$

d. Dibujar  $|A_v(f)|$  Vs  $f$

$$A_v(\omega) = \frac{1}{\sqrt{\left(1 - 2\left(\frac{f}{f_c}\right)^2\right)^2 + \left(2\frac{f}{f_c} - \frac{f^3}{f_c^3}\right)^2}}$$

$$A_v = 20 \log \left( \frac{V_o}{V_i} \right)$$



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<b>f</b>	$ A_v(f) $	$ A_v(f) $ db
<b>0</b>	1	0
<b>0.1fc</b>	1	0
<b>0.5fc</b>	1	0
<b>0.8 fc</b>	0.9	-1
<b>fc</b>	0.7	-3
<b>2 fc</b>	0.1	-18
<b>4 fc</b>	0	-36
<b>6 fc</b>	0	-47
<b>8 fc</b>	0	-54
<b>10 fc</b>	0	-60